Delhi Public School Guwahati
"Under the aegis of the Delhi Public School Society, Delhi"

Practice Questions
Mathematics, Class XII

Chapter: Matrices and Determinants

1.Construct a matrix $A=\left[a_{ij}\right]_{2\times 2}$ whose elements a_{ij} are given by

a) $a_{ij}=e^{2ix}\sin jx$. b) $a_{ij}=\dfrac{(i-2j)^2}{2}$ c) $a_{ij}=\left|-2i+3j\right|$

2. Show that a matrix which is both symmetric and skew symmetric is a zero matrix.

3. If $X=\begin{bmatrix}3 & 1 & -1\\ 5 & -2 & -3\end{bmatrix}$ and $Y=\begin{bmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{bmatrix}$, find a matrix Z such that $X+Y+Z$ is a zero matrix.

4. Find values of a and b if $A=B$ where $A=\begin{bmatrix}a+4 & 3b\\ 8 & -6\end{bmatrix}$ and $B=\begin{bmatrix}2a+2 & b^2+2\\ 8 & b^2-5b\end{bmatrix}$.

5. Find the value of x if $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=O$

6. If $A=\begin{bmatrix}1 & 3 & 2\\ 2 & 0 & -1\\ 1 & 2 & 3\end{bmatrix}$, then show that A satisfies the equation $A^3-4A^2-3A+11I=O$.

7. Let $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$, then show that $A^2-4A+7I=O$. Using this result calculate A^5 also.

8. If $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$, then find $A^2-5A-14I$. Hence obtain A^3.

9. If the matrix $\begin{bmatrix}0 & a & 3\\ 2 & b & -1\\ c & 1 & 0\end{bmatrix}$ is a skew symmetric matrix, find the values of a, b and c.

10. If $P(x)=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$, then show that $P(x).P(y)=P(x+y)=P(y).P(x)$.

11. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ and $A^{-1}=A'$, find the value of α.

12. Find the matrix A satisfying the following equations:

a) $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

b) $\begin{bmatrix}4\\ 1\\ 3\end{bmatrix}A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$

c) $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$

13. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then show that $A^2=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

14. If $A=\begin{bmatrix}1 & 5\\ 7 & 12\end{bmatrix}$ and $B=\begin{bmatrix}9 & 1\\ 7 & 8\end{bmatrix}$, find a matrix C such that $3A+5B+2C$ is a null matrix

15. Show that A′A and AA′ are both symmetric matrices for any matrix A.

16. Express the following matrices as sum of a symmetric and skew-symmetric matrices

a) $\begin{bmatrix}2 & 3 & 1\\ 1 & -1 & 2\\ 4 & 1 & 2\end{bmatrix}$ b) $\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$

17. Give an example of matrices A, B and C such that AB = AC, where A is nonzero matrix, but B ≠ C.

18. Show by an example that for A ≠ O, B ≠ O, AB = O.

19. Find inverse of the following matrices, if exists.

a) $\begin{bmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{bmatrix}$ b) $\begin{bmatrix}2 & 3 & -3\\ -1 & -2 & 2\\ 1 & 1 & -1\end{bmatrix}$ c) $\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$

20. Evaluate a) $\begin{vmatrix}a+ib & c+id\\ -c+id & a-ib\end{vmatrix}$. b) $\begin{vmatrix}\cos 15° & \sin 15°\\ \sin 75° & \cos 75°\end{vmatrix}$ c) $\begin{bmatrix}1 & -3 & 2\\ 4 & -1 & 2\\ 3 & 5 & 2\end{bmatrix}$ d) $\begin{bmatrix}0 & 2 & 6\\ 1 & 5 & 0\\ 3 & 7 & 1\end{bmatrix}$

21. Find x if $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\ 4 & 1\end{vmatrix}$

22. Find the value of k such that the points are collinear

a) A(− 3, 7) , B(7, k) and (2, 1).

b) A(1, −5) , B(−4, 5) and (k, 7).

23. Find the area of the triangle whose vertices are A (11, 7), B (5, 5) and C (-1, 3)

24. Compute A^{-1} for the matrix $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

Hence solve the system of equations:
$$y+2z+8=0$$
$$x+2y+3z+14=0$$
$$3x+y+z+8$$

25. Find A^{-1} for the matrix $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ and show that $A^{-1}=\dfrac{A^2-3I}{2}$

26. Using matrix method solve the following system of equations:

a) $6x-9y-20z=-4$, $4x-15y+10z=-1$, $2x-3y-5z=-1$

b) $2x+y+z=1$, $x-2y-z=\dfrac{3}{2}$, $3y-5z=9$

c) $3x+2y-2z=3$, $x+2y+3z=6$, $2x-y+z=2$

27. If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$x-2y=10,\ 2x-y-z=8,\ -2y+z=7$.

28. Use product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to sole the system of equations $x-y+2z=1,\ 2y-3z=1,\ 3x-2y+4z=2$

29. Given $A=\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}$, find BA and use this to solve the system of equations $y+2z=7,\ x-y=3,\ 2x+3y+4z=17$

30. Prove that $(A^{-1})'=(A')^{-1}$, where A is an invertible matrix.

31. Show that the points $(a+5,\ a-4)$, $(a-2,\ a+3)$ and $(a,\ a)$ do not lie on a straight line for any value of a.

32. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

33. If A and B are invertible matrices, then prove that $(AB)^{-1}=B^{-1}A^{-1}$.

34. If $A=\begin{bmatrix}3 & 1\\2 & -3\end{bmatrix}$, find $|adj\ A|$ and $|A\,adj\ A|$.

35. If $A=\begin{vmatrix}1 & -2 & 3\\0 & -1 & 4\\-2 & 2 & 1\end{vmatrix}$, find $\left(A^T\right)'$. (Ans: $\begin{bmatrix}-9 & -8 & -2\\8 & 7 & 2\\-5 & -4 & -1\end{bmatrix}$)

Chapter: Relations and Functions

36. Let $A=\{1,2,3,........9\}$ and R be the relation in A × A defined by $(a,b)R(c,d)$ if $a+d=b+c$ for $(a,b),(c,d)\in A\times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2,5)]$ and $[(1,3)]$.

37. Show that the relation R on the set Z of all integers defined by $(x,y)\in R \Leftrightarrow (x-y)$ is divisible by 3 is an equivalence relation.

38. Let N be the set of all natural numbers and let R be a relation on N × N, defined by $(a,b)R(c,d)\Leftrightarrow ad=bc$ for all $(a,b),(c,d)\in N\times N$. Show that R is an equivalence relation. Also, find the equivalence class $[(2,6)]$.

39. Let N be the set of all natural numbers and let R be a relation on N × N, defined by $(a,b)R(c,d)\Leftrightarrow ad(b+c)=bc(a+d)$ for all $(a,b),(c,d)\in N\times N$. Show that R is an equivalence relation. Also, find the equivalence class $[(2,6)]$.

40. Let R be the equivalence relation in the set $A=\{0,1,2,3,4,5\}$ given by $R=\{(a,b): 2\ divides\ (a-b)\}$. Write the equivalence class $[0]$.

41. Let $A=[-1,1]$. Then discuss whether the following functions defined on A are one-one onto or bijective.

a) $f(x)=\dfrac{x}{2}$ b) $g(x)=|x|$ c) $h(x)=x|x|$ d) $k(x)=x^2$

42. Check whether following functions are one-one onto or not?

(i) $f(x)=\dfrac{x}{x^2+1}$, $f:R\to R$ (ii) $f(x)=\cos x$ iii) $f(x)=9x^2+6x-5$, $f:R_+\to[-5,\infty)$

iv) $f(x)=5x^2+6x-9$, $f:R_+\to[-9,\infty)$ (R_+ is the set of all non-negative real numbers)

v) $f(x)=4x^2+12x+15$, $f:N\to S$ where S is the range of S

43. Show that $f:N\to N$ given by $f(x)=\begin{cases} x+1, & if\ \ x\ is\ odd \\ x-1, & if\ \ x\ is\ even \end{cases}$ is both one-one and onto.

44. Find the number of all one-one functions from set $A=\{a,b,c\}$ to itself.

Chapter: Inverse Trigonometric Functions

45. Find the principal value of $\tan^{-1}\left(\tan\dfrac{7\pi}{6}\right)$ (Ans: $\dfrac{\pi}{6}$)

46. Find the principal values of i) $\tan^{-1}\left(\tan\dfrac{9\pi}{8}\right)$ ii) $\cot^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$ iii) $\sec^{-1}\left(\sec\dfrac{9\pi}{5}\right)$

47. Evaluate: $\sin\left(\cot^{-1}\left(\cot\dfrac{17\pi}{3}\right)\right)$. (Ans: $\dfrac{\sqrt{3}}{2}$)

48. Find the domain of the following functions:

a) $\sin x+\sin^{-1}x$ b) $\cos^{-1}(3x-2)$

49. Evaluate: i) $\sin^{-1}(\sin 10)$ ii) $\sin^{-1}(\sin 5)$ iii) $\cos^{-1}(\cos 10)$

50. Find the principal value of $\cot^{-1}\left(-\sqrt{3}\right)+\tan^{-1}(1)+\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$. (Ans: $\dfrac{5\pi}{4}$)

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