



SUMMER HOLIDAY HOMEWORK

CHAPTER 1- SETS

- If $X = \{1, 2, 3\}$ and n represents any member of X , write the following sets containing all numbers represented by :
(a) $4n$ (b) $n + 6$ (c) $\frac{n}{2}$ (d) $n - 1$
- Write all possible subsets of the set $\{1, \{1\}\}$
- State and prove the De Morgan's Laws.
- If $a \in N$, such that $aN = \{ax : x \in N\}$, then describe the set $3N \cap 7N$.
- If A and B are any two sets, then prove that $A \subset B \Leftrightarrow B' \subset A'$.
- If A and B are any two sets, then prove that $A - (A - B) = A \cap B$
- For any two sets A and B , prove that $P(A \cap B) = P(A) \cap P(B)$.
- If A, B and C are three sets, then prove that $A - (B \cup C) = (A - B) \cap (A - C)$.
- If A, B and C are three sets, then prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$.
- In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali? How many can speak both Hindi and Bengali?
- A survey of 500 television viewers produced the following information:
285 watch football, 195 watch hockey, 115 watch basketball. 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How many watch all the three games? How many watch exactly one of the three games?
- If A and B are two sets such that $n(A) = 35, n(B) = 30$ and $n(U) = 50$, then find:
(i) the greatest value of $n(A \cup B)$
(ii) the least value of $n(A \cap B)$
- Let Z be the set of all integers and $A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$ and $B = \{(a, b) : a > b, a, b \in Z\}$, then find $A \cap B$.
- Let $A_1, A_2, A_3, \dots, A_{30}$ are thirty sets each with five elements and B_1, B_2, \dots, B_n are n sets each with three elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^n B_j = S$. Assume that each element of S belongs to exactly ten of the A_i 's and exactly 9 of B_j 's. Find n .

CHAPTER 2- RELATIONS AND FUNCTIONS

- If $n(A) = p$ and $n(B) = q$, then find the total number of non-empty relations from A to B .
- If $A = \{1, 3, 5\}$ and $B = \{x, y\}$, then represent $A \times A$ and $A \times B$ by arrow diagrams.
- Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$
Find:
(i) $(A \times B) \cap (A \times C)$ (ii) $A \times (B \cup C)$.
- Determine the domain and range of the following relations:
(i) $R = \{(a, b) : a \in N, a < 5, b = 4\}$
(ii) $S = \{(a, b) : b = |a - 1|, a \in Z \text{ and } |a| \leq 3\}$

5. Let R be the relation on the set N of natural numbers defined by
 $R = \{(a, b): a + 3b = 12, a \in N, b \in N\}$
 Find (i) R (ii) Domain of R (iii) Range of R
6. Determine the domain and range of the following relation:
 $R = \{(a, b): b = |a - 1|, a \in Z \text{ and } |a| \leq 3\}$
7. Let $A = \{1, 2, 3, \dots, 14\}$. Define a relation on set A by
 $R = \{(x, y): 3x - y = 0, \text{ where } x, y \in A\}$
 Depict the relation using an arrow diagram. Write its domain, co domain and range.
8. If a function $f: R \rightarrow R$ be defined by

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

 Find $f(1), f(-1), f(0), f(2)$.
9. Find the domain and range of the following functions:
 (i) $f(x) = \frac{x}{1+x^2}$
 (ii) $f(x) = \frac{2x+1}{x^2-9}$
 (iii) $f(x) = \frac{1}{\sqrt{x^2-1}}$
 (iv) $f(x) = |x - 1|$
10. Let $f(x) = 2x + 5$ and $g(x) = x^2 + x$.
 Describe (i) $f+g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$.
 Also, find the domain in each case.
